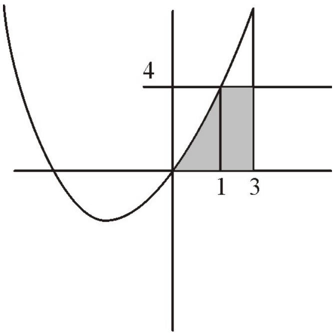
48.(1)



Required area $= \int_0^1 (x^2 + 3x)dx + 4 \times 2 = \frac{11}{6} + 8 = \frac{59}{6}$

49.(3) $g(f(x)) = \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$; $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx$; $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\frac{2 + x\cos x}{2 - x\cos x}\right)dx$

$\Rightarrow \quad 2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(1)dx = 0 \quad \Rightarrow \quad I = 0 \quad \Rightarrow \quad \log_e 1$

50.(1) $f(x) = \int_0^x g(t)dt$

$g(x)$ is even $fn. \therefore$ $f(x)$ is odd fn.

Also $f'(x) = g(x)$

$f(x+5) = g(x)$; $f(5-x) = g(-x) = g(x) = f(x+5)$

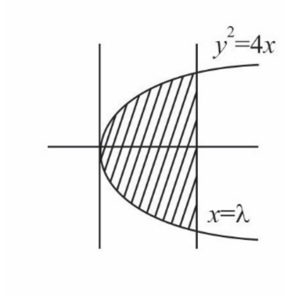
Now $\int_0^x f(t)dt$

$t = 5 + z \qquad dt = dz$

$\int_{-5}^{x-5} f(5+z)dz = \int_{-5}^{x-5} g(z)dz$

$= \int_{-5}^{x-5} f'(z)dz = f(x-5) - f(-5) = f(5) - f(5-x) = f(5) - f(5+x) = \int_{x+5}^{5} f'(t)dt = \int_{x+5}^{5} g(t)dt$

51.(1) $y^2 = 4x$



$$S(I) = 2\int_0^{\lambda} 2\sqrt{x}\,dx = \left.\frac{4x^{3/2}}{3/2}\right]_0^{\lambda} = \frac{8}{3}\lambda^{3/2}$$

$$\frac{S(\lambda)}{S(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5} \quad ; \quad \lambda = 4\left(\frac{2}{5}\right)^{2/3} = 4\left(\frac{4}{25}\right)^{1/3}$$

52.(3) $\int_0^{2} \frac{\sin^3 x}{\sin x + cosx}\,dx = I\,(say) \qquad \Rightarrow \qquad I = \int_0^{\pi/2} \frac{\cos^3 x}{\sin x + \cos x}dx$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin x + cosx}dx \qquad \Rightarrow \quad 2I = \int_0^{\pi/2} (\sin^2 x + \cos^2 x - \sin x \cos x)dx$$

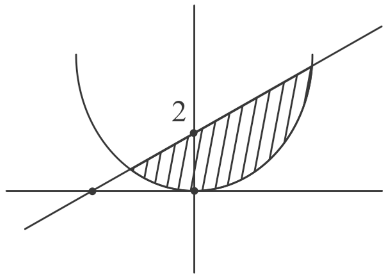
$$\Rightarrow \quad 2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin 2x\right)dx \qquad \Rightarrow \quad 2I = x + \left.\frac{\cos 2x}{4}\right]_0^{\pi/2}$$

$$\Rightarrow \quad 2I = \left(\frac{\pi}{2} - \frac{1}{4}\right) - \left(\frac{1}{4}\right) = \frac{\pi}{2} - \frac{1}{2} \qquad \Rightarrow \quad I = \frac{\pi - 1}{4}$$

53.(2) $A = \left\{(\lambda, 4);\, x^2 \le y \le x + 2\right\}$

$y \ge x^2$ and $y - x - 2 \le 0$

Point of intersection



$$\Rightarrow \quad x^2 = x + 2 \quad \Rightarrow \quad x^2 - x - 2 = 0$$

$$x = -1, 2$$

$$A = \left[\int_{-1}^{2} \left((x+2) - x^2\right)dx = \frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} = \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{9}{2}$$

54.(1) $I = \int_0^1 x \cdot \cot^{-1}\left(1 - x^2 + x^4\right)dx$

Put $\quad x^2 = t$

$2xdx = dt$

$$I = \frac{1}{2}\int_0^1 \cot^{-1}\left(1 - t + t^2\right)dt = \frac{1}{2}\int_0^1 \tan^{-1}\left(\frac{1}{1 + t(t-1)}\right)dt = \frac{1}{2}\int_0^1 \tan^{-1}\left(\frac{t - (t-1)}{1 + t\cdot(t-1)}\right)dt$$

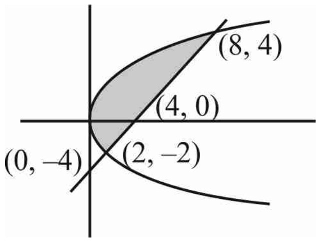
$$= \frac{1}{2}\int_0^1 \left\{\tan^{-1} t + \tan^{-1}(1-t)\right\}dt \qquad \therefore \qquad \int_0^1 \tan^{-1} t\; dt = \int_0^1 \tan^{-1}(1-t)dt$$

$$I = \int_0^1 \tan^{-1} t\; dt \qquad = t\cdot\tan^{-1} t\Big|_0^1 - \int_0^1 \frac{t}{1+t^2}dt = \frac{\pi}{4} - \frac{1}{2}\ell n\left(1 + t^2\right)\Big|_0^1 = \frac{\pi}{4} - \frac{1}{2}\ell n 2 \text{ Ans.}$$

55.(1) $y^2 \le 2x,\ x \le y+4$

Solve $y^2 = 2x$ and $x = y+4$; we get (8, 4) & (2, –2)

Shaded area $= \int_{-2}^{4}\left(\left(y+4\right)-\frac{y^2}{2}\right)dy = 18$



56.(1) $\int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$

Let $I = \int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$ ... (1)

$I = \int_0^{2\pi}\left[\sin 2(2\pi - x)(1+\cos 3(2\pi - x))\right]dx$

$I = \int_0^{2\pi}\left[-\sin 2x(1+\cos 3x)\right]dx$ ... (ii)

(1) + (2)

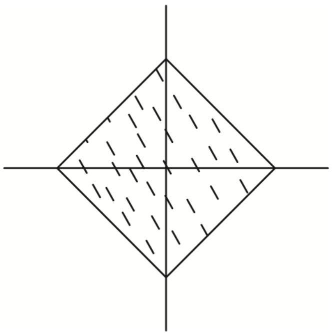
$2I = \int_0^{2\pi}(-1)dx$ $\quad [x]+[-x] = -1$

$2z = -2\pi \quad \Rightarrow \quad I = -\pi$

57.(2) Here it is given that $|x-y| \le 2$ ... (1)

and $|x+4| \le 2$ ... (2)

Combinating 2



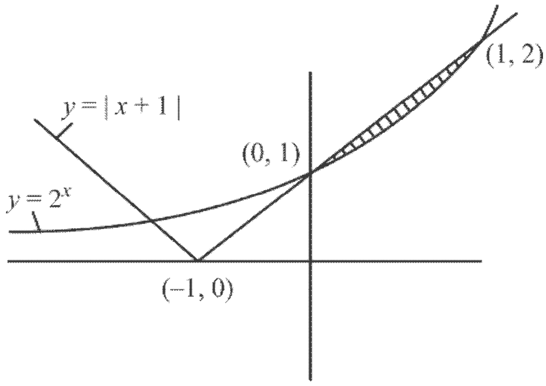
Square of side length $2\sqrt{2}$

58.(2) $\because \quad I = \int_{\pi/6}^{\pi/3}\frac{dx}{\cos^{2/3}x.\sin^{4/3}x} = \int_{\pi/6}^{\pi/3}\frac{\sec^2 x\ dx}{\tan^{4/3}x}$

Put $\tan x = t \Rightarrow \sec^2 x\ dx = dt$

When $x = \frac{\pi}{6}, t = \frac{1}{\sqrt{3}}$; when $x = \frac{\pi}{3}, t = \sqrt{3} \quad \therefore \quad I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}\frac{dt}{t^{4/3}} = -3t^{-1/3}\Big]_{1/\sqrt{3}}^{\sqrt{3}} = 3^{\frac{7}{6}} - 3^{\frac{5}{6}}.$

59.(1)



$$A = \int_0^1 (x+1) - 2^x\,dx = \frac{3}{2} - \frac{1}{\ln 2}$$

60.(4) $\displaystyle\int_0^{\pi/2} \frac{\cot x}{\cot x + \text{cosex}} = \int_0^{\pi/2} \frac{\cos x}{1+\cos x}dx \int_0^{\pi/2} dx^- \int_0^{\pi/2} \frac{1}{1+\cos x}dx$

$$= \left[x\right]_0^{\pi/2} - \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{\pi}{2}dx = \frac{\pi}{2} - \frac{1}{2}\left|\frac{\tan\frac{\pi}{2}}{\frac{1}{2}}\right|_0^{\pi/2} = \frac{\pi}{2} - 1 \quad \Rightarrow \quad \frac{1}{2}(\pi - 2) \quad \Rightarrow \quad M = \frac{1}{2}, n = 2$$

$\therefore \qquad MN = -1$

61.(4) $\displaystyle A = \int_0^{2\sqrt{2}-2} \left(1 - y - \frac{y^2}{4}\right)dy$

$$A = \left[y - \frac{y^2}{2} - \frac{y^3}{12}\right]_0^{2\sqrt{2}-2} = y\left[1 - \frac{y}{2} - \frac{y^2}{12}\right]$$

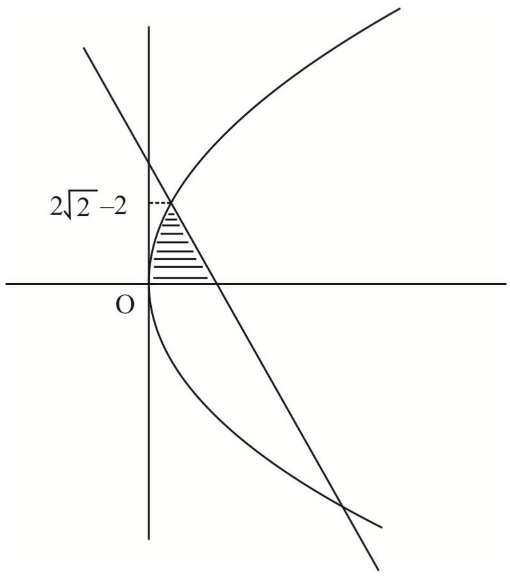
$$= 2\left(\sqrt{2}-1\right)\left[1 - \left(\sqrt{2}-1\right) - \frac{4}{12}\left(\sqrt{2}-1\right)^2\right]$$

$$= 2\left(\sqrt{2}-1\right)\left[2 - \sqrt{2} - \frac{1}{3}\left(2+1-2\sqrt{2}\right)\right]$$

$$= 2\left(\sqrt{2}-1\right)\left[2 - \sqrt{2} - 1 + \frac{2\sqrt{2}}{3}\right]$$

$$= 2\left(\sqrt{2}-1\right)\left[1 - \frac{\sqrt{2}}{3}\right] = 2\left(\sqrt{2} - \frac{2}{3} = 1 + \frac{\sqrt{2}}{3}\right)$$

$$= 2\left(\frac{4}{3}\sqrt{2} - \frac{5}{3}\right)$$



62.(2) $\displaystyle\int_0^{f(x)} 4t^3 dt = (x-2)g(x)$

Differentiating both sides w.r.t x

$4\left(f(x)\right)^3 \times f'(x) = (x-2)g'(x) + g(x)$

$\displaystyle\lim_{x\to 2} 4 \times 6 \times \frac{1}{48} = 9(2) \Rightarrow 18$

63.(4) $\displaystyle\int_\alpha^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \int_\alpha^{\alpha+1} \left(\frac{1}{(x+\alpha)} - \frac{1}{(x+\alpha+1)}\right)$

$\displaystyle = \ln\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg]_\alpha^{\alpha+1} \quad = \ln\left(\frac{2\alpha+1}{2\alpha+2}\right) - \ln\left(\frac{2\alpha}{2\alpha+1}\right) \quad = \ln\left(\frac{(2\alpha+1)^2}{2(\alpha)(\alpha+1)}\right) = \ln\frac{9}{8} \quad \Rightarrow \quad \alpha = -2$

64.(2) $y^2 = 4\lambda x,\ y = \lambda x$

$\displaystyle A = \int_0^{4/\lambda} (2\sqrt{\lambda x} - \lambda x)\, dx \quad \Rightarrow \quad A = \frac{\frac{2\sqrt{\lambda}\,8}{\lambda^{3/2}}}{\frac{3}{2}} - \frac{\lambda \times \frac{16}{\lambda^2}}{2} = \frac{1}{9} \quad \Rightarrow \quad \lambda = 24$